Low-Complexity Adaptive Channel Estimation for MIMO-OFDM Systems over Rayleigh FadingChannels

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Abstract: *In this paper, we propose novel low-complexity adaptive scheme infrequency domain for estimation of MIMO-OFDM channels.Computational complexity of recursive least square (RLS)estimation algorithm is reduced through QR decomposition (QRD) andinverse QR decomposition (IQRD) methods. Simulation results demonstrate the usefulness of the proposed scheme for MIMO-OFDMchannel estimation.*

Keywords: Adaptive filter, channel estimation, MIMO-OFDM systems, low-complexity.

1. Introduction

Multiple-input multiple-output (MIMO) antenna architecture has the ability to increase capacity and reliability of a wireless communication system. On the other hand, orthogonal frequency-division multiplexing (OFDM) technology is well-known for efficient high speed transmission and robustness to frequency selective channels. Hence, the integration of the two technologies has the potential to meet the ever growing demands of future communication systems [1], [2]. However, accurate channel state estimation is important in communication systems and the performance of these systems is their reliance upon the availability of accurate channel state information (CSI) at the receiver. When the channel varies from block to block adaptive filtering techniques are suitable for estimation of such time variant channel state information.

There are many adaptive filter algorithms that are widely used for channel estimation, but they either have a high mean-square error with slow convergence rate (such as, least mean square (LMS) and normalized least mean square (NLMS) algorithms) or a high computation complexity with fast convergence rate and low mean-square error (such as, recursive least square (RLS) algorithm). Thus, mean-square error (MSE), convergence rate and computational complexity are three important points that should be considered in selecting of the adaptive algorithms for channel estimation [3], [4], [5]. The RLS method has good performance, but it has the problem of numerical instability. This motivates the application of stable orthogonal linear transformations to the original RLS problem. In [3], it states that the QRD is a numerically stable method of solving the least square (LS) estimation. Moreover, numerous researchers have observed that the time-recursive filtering problem exhibits more stable properties when implemented in the QR decomposition form [3], [6], [7].

Another important benefit of QR-based approaches is that the rotations computations are easily mapped onto systolic array structures for a parallel implementation [3], [6]. In this paper, QRD and IQRD will be applied to solve the RLS problem and these adaptive algorithms will be used for estimation of MIMO-OFDM channels in frequency domain. In the QR decomposition recursive least squares (QRD-RLS) algorithm, triangular transformation using Givens rotations is used to recursively solve the LS problem. The adaptive filtering performed directly to the data matrix as oppose to the RLS method, which works on the time averaged correlation matrix of the input data.

The main contributions of this work are as follows.

- First, we propose novel adaptive approach for estimationof MIMO-OFDM channels using pilot signals.
- Develop LMS and RLS algorithms in frequency domain forMIMO-OFDM systems.
- Extension of the QRD and IQRD methods to solving the recursive least square estimation and the establishment of low-complexity adaptive algorithms. These methods cause the computational complexityof the proposed algorithms to be reduced to one-half thenumerical dynamic range of the original RLS computational problem.
- Finally, we use all of these algorithms for channel estimation. The main goal of this paper is present a low-complexity adaptive estimator for MIMO-OFDM systems over multipath Rayleigh fading channels.

The rest of this paper is organized as follows. In Section 2, we will have a channel model and problem statement.

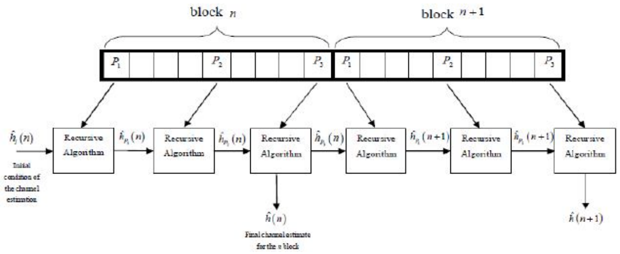

Fig. 1. Adaptive scheme for channel estimation in frequency domain using pilot signals

Subsequently, adaptive scheme for MIMO-OFDM channel estimation in frequency domain will be presented in Section 3. Section 4 describes the QR decomposition recursive least square method. In the next section, the inverse QR decomposition recursive least square method is presented. Finally, Section 6 demonstrates the performance of the proposed estimation approaches by computer simulation result. Section 7 concludes this paper.

2. Channel Model and Problem Statement

Consider the MIMO-OFDM system consisting of M_t transmit and $M_r$receive antennas operating in a Rayleigh frequencyselective fading environment. Using the tap-delay channel model ofthe multiple channels, the channel estimation problem can beformulated as that of an adaptive filtering problem. Let$\boldsymbol{S}_t^i$be the frequency domain input block of Ncomplex-valued symbol prior to OFDM processing at time$t$ at thei^{th} transmit antenna, where$\boldsymbol{S}_t^i = [S_t^i\,(0), S_t^i\,(1), \dots, S_t^i\,(N-1)]^H, i = 1,2, \dots, M_t$ and $\boldsymbol{s}_t^i$is the time domain representation after inverse fastFourier transform (IFFT) operation with N_{CP} cyclic prefix(CP) symbols inserted, where $\boldsymbol{s}_t^i = [s_t^i\,(0), s_t^i\,(1), \dots, s_t^i\,(N-1)]H, i=1,2,\dots, Mt$.

In matrix form, the time domain received signal before OFDMdemodulation can be expressed as

$$\boldsymbol{y}_t = \boldsymbol{G}\boldsymbol{s}_t + \boldsymbol{v}_t, \;\; t = 0, 1, 2, \dots \qquad (1)$$

Where $\boldsymbol{y}_t = [\boldsymbol{y}_t(0), \boldsymbol{y}_t(1), \dots, \boldsymbol{y}_t(N + N_{cp} - 1)]^H$ and $\boldsymbol{v}_t = [\boldsymbol{v}_t(0), \boldsymbol{v}_t(1), \dots, \boldsymbol{v}_t(N + N_{cp} - 1)]^H$ are $(M_r(N + N_{cp}) \times 1)$ vectors, $\boldsymbol{s}_t = [\boldsymbol{s}_t(0), \boldsymbol{s}_t(1), \dots, \boldsymbol{s}_t(N - l)]^H$ is a vector of size $(M_r N \times 1), l$ is channel length (for $0 \le l \le L-1$) and

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{h}(0) & & & \\ \boldsymbol{h}(1) & \boldsymbol{h}(0) & & \\ \vdots & \boldsymbol{h}(1) & & \\ \boldsymbol{h}(L-1) & \vdots & \ddots & \\ & \boldsymbol{h}(L-1) & \ddots \;\; \ddots & \\ & & \ddots & \boldsymbol{h}(0) \\ & & \ddots & \boldsymbol{h}(1) \\ & & \ddots & \vdots \\ & & & \boldsymbol{h}(L-1) \end{bmatrix}$$

is a $(M_r(N + N_{cp}) \times M_t N)$ matrix. After $\boldsymbol{y}_t$ is OFDM demodulated by the fast Fourier transformation (FFT) and the CP is removed, the received signal for all the receive antennas in the frequency domain can be written as

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{H} + \boldsymbol{V} \qquad (2)$$

Where $\boldsymbol{Y}$and $\boldsymbol{V}$are the received signaland the noise matrixes of size$(N \times M_r)$, respectively. $\boldsymbol{H}$ is the channel frequency response matrix withdimensions$(NM_t \times M_r)$ and $\boldsymbol{X}$is a$(N \times NM_t)$ matrix of the transmitted signal. Note that thetransmitted signal at i^{th} transmit antenna and j^{th}receive antenna is a diagonal matrix with dimensions$(N \times N)$.

$(\boldsymbol{X}^{(j,i)} = diag\{S_0^{(j,i)}, S_1^{(j,i)}, \dots, S_{N-1}^{(j,i)}\})$

Define the $N \times L$ matrix $[F]_{n,l} := exp(j2\pi(n-1)(l-1)/N)$and let $\boldsymbol{\Phi} = diag\{F, F, \dots, F\}$ be a blockdiagonal matrix of F. Then, considering this definition equation (2) can be rewritten in term of the channel impulseresponse (CIR) as follows

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\phi}\boldsymbol{h} + \boldsymbol{V} = \boldsymbol{W}\boldsymbol{h} + \boldsymbol{V} \qquad (3)$$

It has been shown in [8] that using the received signalmodel in above equation the LS estimate is$\widehat{\boldsymbol{h}} = (\boldsymbol{W}^H\boldsymbol{W})^{-1}\boldsymbol{W}^H\boldsymbol{Y}$. In a time variant channel, the channel changesover time, and it is not practical to calculate the LS estimate atevery block. Therefore, recursive adaptive algorithms are usedwhere the LS solution is not explicitly calculated.

3. Adaptive Scheme for MIMO-OFDM Channel Estimation in Frequency Domain

The application of adaptive filtering for channel estimation inMIMO-OFDM systems is more complex due to the multiple channels. Toadaptively adjust the estimates of each multipath channel requiresthe use of multichannel adaptive filtering techniques. Inmultichannel filtering the error between the received signal andthe sum of each channel output is used to adjust the channelestimates.

In [9], a fast multichannel QR decomposition leastsquares adaptive algorithm was introduced to solve for the CIR. Inthe case of MIMO-OFDM, we have established that it is moreconvenient to process the received signal in the frequency domain.The received signal in the frequency domain can be represented asa multiplication of the channel frequency response and thetransmitted signal. In the frequency domain, the objective ofadaptive filtering is to use the values of the previous subcarrierto estimate the current subcarrier. In the recursive approach, thek^{th} inputs and outputs, and the previous value of the channel $h(k-1)$ are used to estimate $h(k)$. To better illustrate thisidea, Fig. 1 shows the structure of the recursive channelestimation method. Basically, at each pilot subcarrier an updatedchannel estimate is obtained using the previous channel estimatesand the current pilot information. As the diagram shows, a newestimate is obtained at each pilot subcarrier. It should be notedthat it is valid to use the last estimate of the previous block as initial conditions of the current block, since the channel doesnot change drastically between adjacent blocks.

Considering equation (3), the received signal of a singlesubcarrier is given as

$$\boldsymbol{Y}(k) = \boldsymbol{W}(k)\boldsymbol{h} + \boldsymbol{V}(k) \quad (4)$$

RLS uses the technique of least square filtering with a costfunction defined as the sum of weighted error squares:

$$\boldsymbol{J}(k) = \sum_{k=1}^{n} \lambda^{n-k} (\boldsymbol{Y}(k) - \boldsymbol{W}(k)\boldsymbol{h})^H (\boldsymbol{Y}(k) - \boldsymbol{W}(k)\boldsymbol{h}) \quad (5)$$

Where $\boldsymbol{h}$ is the CIR matrix to be estimated,$\lambda$is the forgetting factor that is a positive constantless that unity and n is the variable length of the observationdata. With the known (pilot) vector,$\boldsymbol{W}(k)$, and the received signal, $\boldsymbol{Y}(k)$, where $k = 1,2,\ldots,Np$, the exponentially weighted RLS algorithm described in [4] canbe used to adaptively track the CIR matrix. The algorithm issummarized in Table I.

In Table I, the inverse correlation matrix,$\boldsymbol{P}(k)$, is of size $(LM_t \times LM_t)$ and the Kalman gain vector, $\boldsymbol{G}(k)$, is of size $(LM_t \times 1)$. The RLS method has good performance, but it has theproblem of numerical instability. In the following sections, QRDand IQRD will be applied to solve the RLS problem.

TABLE I: SUMMARY OF RLS ALGORITHM F OR MIMO-OFDM CHANNEL ESTIMATION

$\boldsymbol{P}(0) = \delta^{-1}\boldsymbol{I}$, where δ is a small positive constant (i.e. 0.001)
$\widehat{\boldsymbol{h}}(0) = 0_{M_r \times LM_t}$
For each pilot vector $\boldsymbol{W}(k), k = 1,2,\ldots,N_P$, compute
$\boldsymbol{T}(k) = \lambda^{-1}\boldsymbol{W}(k)\boldsymbol{P}(k-1)/1 + \lambda^{-1}\boldsymbol{P}(k-1)\boldsymbol{W}^H(k)$
$\boldsymbol{e}(k) = \boldsymbol{Y}(k) - \boldsymbol{W}(k)\widehat{\boldsymbol{h}}(k-1)$
$\widehat{\boldsymbol{h}}(k) = \widehat{\boldsymbol{h}}(k-1) + \boldsymbol{T}^H(k)\boldsymbol{e}(k)$
$\boldsymbol{P}(k) = \lambda^{-1}\boldsymbol{P}(k-1) - \lambda^{-1}\boldsymbol{T}^H(k)\boldsymbol{W}(k)\boldsymbol{P}(k-1)$

4. QRD RECURSIVE LEAST SQUARE METHOD

To develop the QRD-RLS algorithm for channel estimation inMIMOOFDM systems the RLS cost function described in (5)needs to be rewritten in matrix-vector notation. Assuming apre-windowing of Np samples of the input data, the data matrixat time n is defined as following [shown at the top of the nextpage]and the forgotten factor matrix is

$$\boldsymbol{\Lambda}(n) = diag\,\{\sqrt{\lambda^{n-N_p}}, \sqrt{\lambda^{n-N_p+1}}, \ldots.,1\} \quad (7)$$

The cumulative squared error can be written in matrix form as

$$J(n) = \|\boldsymbol{\Lambda}(n)\boldsymbol{Y}(n) - \boldsymbol{\Lambda}(n)\boldsymbol{W}(n)\boldsymbol{h}(n)\|^2 \quad (8)$$

Now, we apply QRD to$\boldsymbol{\Lambda}(n)\boldsymbol{W}(n)$ to get

$$\boldsymbol{Q}^H(n)\boldsymbol{\Lambda}(n)\boldsymbol{W}(n) = \begin{bmatrix}\boldsymbol{R}(n)\\ 0\end{bmatrix} \quad (9)$$

Where $\boldsymbol{Q}^H(n)$ is the complex transpose of the orthogonal matrix of size$N_p \times N_p$, and $R(n)$ is a $M_tL \times M_tL$upper triangular matrix. The same orthogonal matrix is applied to$\boldsymbol{\Lambda}(n)\boldsymbol{Y}(n)$ to give

$$\boldsymbol{Q}^H(n)\boldsymbol{\Lambda}(n)\boldsymbol{Y}(n) = \begin{bmatrix}\boldsymbol{Z}(n)\\ \boldsymbol{\Delta}(n)\end{bmatrix} \quad (10)$$

Where $\boldsymbol{z}(n)$ is a $M_tL \times M_r$ matrix and $\boldsymbol{\Delta}(n)$ is $(N_P - M_tL) \times M_r$ matrix. Since $\boldsymbol{Q}(n)$is orthogonal, it can premultiply eachvector within the norm without altering the value of the norm.Substituting (9) and (10) into the cost function (8) we get

$$J(n) = \|\boldsymbol{Q}(n)\boldsymbol{\Lambda}(n)\boldsymbol{Y}(n) - \boldsymbol{Q}(n)\boldsymbol{\Lambda}(n)\boldsymbol{W}(n)\boldsymbol{h}(n)\|^2 = \left\|\begin{matrix}\boldsymbol{Z}(n) - \boldsymbol{R}(n)\boldsymbol{h}(n)\\ \boldsymbol{\Delta}(n)\end{matrix}\right\| \quad (11)$$

Minimization of the above function gives

$$\boldsymbol{R}(n)\widehat{\boldsymbol{h}}(n) = \boldsymbol{Z}(n) \quad (12)$$

In recursive algorithms the previous values are assumed to beknown and the new input data are used to solve for the currentvalues. Assume that $\boldsymbol{R}(n-1)$ and $\boldsymbol{Z}(n-1)$are known, then the current values can be obtained by

$$\boldsymbol{B}(n)\begin{bmatrix}\sqrt{\lambda}\boldsymbol{R}(n-1)\\ \boldsymbol{W}(n)\end{bmatrix} = \begin{bmatrix}\boldsymbol{R}(n)\\ 0^T\end{bmatrix} \quad (13)$$

Where $\boldsymbol{B}(n)$ is an orthogonal matrix that zeros the new data vector. Similarly, $\boldsymbol{Z}(n)$ is updated as

$$\boldsymbol{B}(n)\begin{bmatrix}\sqrt{\lambda}\boldsymbol{Z}(n-1)\\ \boldsymbol{Y}(n)\end{bmatrix} = \begin{bmatrix}\boldsymbol{Z}(n)\\ \boldsymbol{\delta}(n)\end{bmatrix} \quad (14)$$

Where $\boldsymbol{Y}(n)$the received signal vector from allthe is received antennas, and $\boldsymbol{\delta}(n)$ is the last rowon the right hand side matrix. With the current values of$\boldsymbol{R}(n)$ and $\boldsymbol{Z}(n)$ the new channelestimates can be easily obtained by back substitution.

5. IQRD RECURSIVE LEAST SQUARE METHOD

The QRD-RLS algorithm is computationally efficient and numerically stable; nonetheless, the back substitution procedure to obtain thechannel estimates adds time delays to the algorithm. In [10], a better alternative was developed to eliminate the need for back substitution by updating the inverse of$\boldsymbol{R}(n)$. In this section the inverse QR decompositionrecursive least squares (IQRD-RLS) algorithm will be adapted to aMIMO-OFDM system. From relation (12), we see that thechannel estimation is given as

$$\hat{h}(n) = R^{-1}(n)Z(n) \quad (15)$$

Assuming that $R^{-1}(n-1)$ is known, the $R^{-1}(n)$ needs to be updated recursively. To determine the orthogonal updating matrix *D*(*n*), we must first define a intermediate matrix

$$\alpha(n) = \frac{R^{-H}(n-1)h(n)}{\sqrt{\lambda}} \quad (16)$$

The orthogonal matrix *D* (*n*) is different from the *B* (*n*) of the QRD-RLS algorithm in that it is not obtained in a straight forward manner through the zeroing of the new data vector by a sequence of Givens rotations. The *D* (*n*) is obtained by performing given rotations to zero the n elements of the augmented $a(n)$ matrix as follows

$$D(n)\begin{bmatrix}\alpha(n)\\ 1^T\end{bmatrix}=\begin{bmatrix}o\\ b(n)\end{bmatrix} \quad (17)$$

Where 1 = [1; 1; :::; 1] and $b(n)$ is the resulting vector of the series of Givens rotations. It turns out that the D (n) obtained in the above equation also satisfies the following equation

$$D(n)\begin{bmatrix}\sqrt{\lambda}R^{-H}(n-1)\\ 0^T\end{bmatrix}=\begin{bmatrix}R^{-H}(n)\\ u^H(n)\end{bmatrix} \quad (18)$$

Where

$$u(n)=\frac{R^{-1}(n-1)a(n)}{\sqrt{\lambda}b(n)} \quad (19)$$

Therefore, using D (n), the R^{-1} (n) matrix can be recursively updated and the channel estimates can be obtained by simply multiplications as shown in (15).

6. Simulation Result

We consider an SISO-OFDM system with $2,048$ subcarrierscorresponding to the binary phase shift keying (BPSK) input signalin terrestrial digital video broadcasting (DVB-T) [11]. The frequency selective channel has $L = 40$ zero-mean uncorrelatedcomplex Gaussian random taps and the number of pilot symbols is$Np = 64$. The step-size and forgetting factor are set equal to$\mu = 0.1$ and $\lambda = 0.99$ for LMS and RLS algorithms,respectively. The signal to noise ratio (SNR) is equal to$30{\sim}dB$.

Fig. 2 depicts the performance of RLS algorithm is better than LMSalgorithm, and RLS, QRD-RLS and IQRD-RLS algorithms have similarperformance. However, the QRD method can be implemented in ahighly parallel systolic array structure which makes it desirablefor real-time implementations. Furthermore, the IQRD-RLS ispreferable since it tracks the inverse correlation matrixtherefore would not require back substitution to solve for the channel matrix.

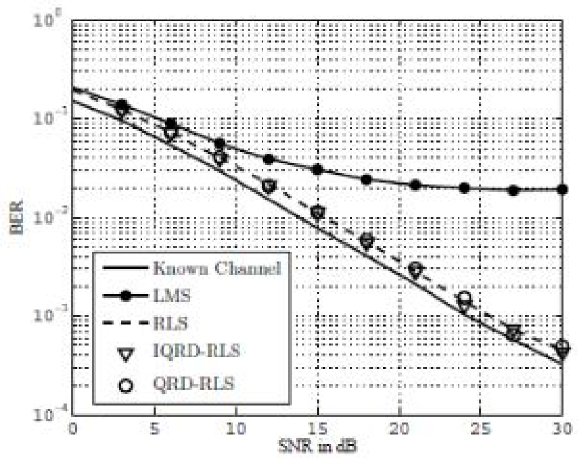

Fig. 2. BER comparison between LMS, RLS, QRD-RLS and IQRD-RLS channel estimation versus SNR.

7. Conclusion

In this paper, we presented new low-complexity adaptive scheme forestimation of MIMO-OFDM channels. We established multichanneladaptive filtering techniques in frequency domain. For reducing computational complexity, the low-complexity QRD and IQRD methodsto solving the recursive least square estimation are proposed inthis paper. The simulation result shows that the application of QRdecomposition greatly reduces the complexity of recursive leastsquare estimation and has a good performance.

Acknowledgements

This work was supported in part by Shahr-e-Rey Branch, Islamic Azad University, Tehran, Iran. The authors gratefully acknowledge for their financial and other supports.

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